



SIAM Conference on
Computational Geometric Design

Program and Abstracts

Simon Fraser University, Vancouver

June 17-19, 2019

SIAM/GD 19

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Simon Fraser University, Vancouver, June 17-19, 2019



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8:30–9:00	Registration HCC	
9:00–10:00	KN #1 JRS 1420–1430	
10:00–10:20	Coffee break HCC	
10:20–11:20	KN #2 JRS 1420–1430	
11:20–12:00		
12:00–13:30	Lunch on site	
13:30–14:50	CP #1 JRS 1420	CP #2 JRS 1430
	MS #1 JRS 1420	CP #3 JRS 1430 (14:50–15:50)
14:50–16:30	Coffee break HCC	
16:30–16:50		
16:30–18:35	MS #1 JRS 1420 (16:50–18:05)	MS #2 JRS 1430

Tue, June 18		
8:30–9:00	Registration HCC	
9:00–10:00	CP #4 JRS 1420	CP #5 JRS 1430
10:00–10:20	Coffee break HCC	
10:20–12:00	MS #3 JRS 1420 (10:20–11:35)	MS #4 JRS 1430
	Lunch on site	
12:00–13:30		
13:30–15:10	MS #5 JRS 1420	MS #6 JRS 1430
15:10–15:30	Coffee break HCC	
15:30–17:10	MS #7 JRS 1420	CP #6 JRS 1430 (15:50–16:30)
	SIAG/GD Business meeting JRS 1420 (17:30–18:30)	MS #8 JRS 1430
16:30–18:35		
17:10–17:30	Coffee break HCC	

HCC = Harbour Center Concourse
JRS = Joseph & Rosalie Segal Centre

Wed, June 19	
8:00–8:30	Registration HCC
8:30–9:30	Summit Keynote #1 from SIAM/GD Djavad Cinema
9:30–9:45	Coffee break HCC
9:45–10:45	Summit Keynote #2 from SMI Djavad Cinema
10:45–11:00	Coffee break HCC
11:00–12:00	Summit Keynote #3 from Bézier Awardee Djavad Cinema
12:00–13:30	Lunch on site
13:30–14:30	Summit Keynote #4 from GMP Djavad Cinema
14:30–15:30	Poster FF (Djavad Cinema, 14:30–15:00) poster display + coffee and refreshments Djavad lobby
15:30–16:20	
16:20–18:00	MS #9 JRS 1420
	CP #7 JRS 1430 (16:20–17:40)
18:00–18:30	SIAM/GD best paper awards JRS 1420
18:30–19:30	
19:30–21:30	Summit dinner banquet

HCC = Harbour Center Concourse
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Keynote Talks

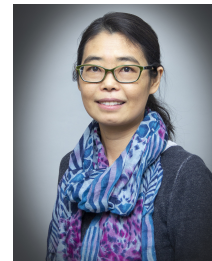
KN1 A Practical Unstructured Spline Modeling Platform for Isogeometric Analysis Applications

Mon, h. 9:00-10:00 in JRS 1420-1430

Chair: Kai Hormann, Università della Svizzera Italiana, Lugano, Switzerland.

Yongjie Jessica Zhang

Professor of Mechanical Engineering,
Courtesy Appointment in Biomedical Engineering,
Carnegie Mellon University, USA.



Biography

Jessica Zhang is a Professor of Mechanical Engineering at Carnegie Mellon University with a courtesy appointment in Biomedical Engineering. She received her B.Eng. in Automotive Engineering, and M.Eng. in Engineering Mechanics from Tsinghua University, China; and M.Eng. in Aerospace Engineering and Engineering Mechanics and Ph.D. in Computational Engineering and Sciences from Institute for Computational Engineering and Sciences (ICES), The University of Texas at Austin. After staying two years at ICES as a postdoctoral fellow, she joined CMU in 2007 as an assistant professor, and then was promoted to an associate professor in 2012 and a full professor in 2016. Her research interests include computational geometry, mesh generation, computer graphics, visualization, finite element method, isogeometric analysis and their application in computational biomedicine, material sciences and engineering. She has co-authored over 170 publications in peer-reviewed journals and conference proceedings, and received the Best Paper Award 1st Place in Solid and Physical Modeling Conference 2018, Autodesk Best Paper Award 1st Place in SIAM Conference on Solid and Physical Modeling 2015, the Best Paper Award in CompIMAGE'16 conference and one of the 5 Most Highly Cited Papers Published in Computer-Aided Design during 2014-2016. She published a book entitled "Geometric Modeling and Mesh Generation from Scanned Images" with CRC Press, Taylor & Francis Group in 2016. She is the recipient of ELATE Fellow

at Drexel, Presidential Early Career Award for Scientists and Engineers, NSF CAREER Award, Office of Naval Research Young Investigator Award, USACM Gallagher Young Investigator Award, Clarence H. Adamson Career Faculty Fellow in Mechanical Engineering, George Tallman Ladd Research Award, and Donald L. & Rhonda Struminger Faculty Fellow. She is also a Fellow of USACM.

Abstract

As a new advancement of traditional finite element method, isogeometric analysis (IGA) adopts the same set of basis functions to represent both the geometry and the solution space, integrating design with analysis seamlessly. In this talk, I will present a practical unstructured spline modeling platform that allows IGA to be incorporated into existing commercial software such as Abaqus and LS-DYNA, heading one step further to bridge the gap between design and analysis. The platform includes all the necessary modules of the design-through-analysis pipeline: pre-processing, surface and volumetric spline construction, analysis and post-processing. Taking IGES files from commercial computer aided design packages, Rhino specific files or mesh data, the platform provides several control mesh generation techniques, such as converting any unstructured quadrilateral/hexahedral meshes to T-meshes, frame field based quadrilateral meshing, and polycube method. Truncated T-splines, hierarchical B-splines and subdivision basis functions are developed, supporting efficient local refinement and sharp feature preservation. To ensure analysis suitability, partition of unity, linear independence and optimal convergence rate of these basis functions are studied in our research. In the end, several practical application problems are presented to demonstrate the capability of our software platform.

References:

- [1] Y. Lai, Y. J. Zhang, L. Liu, X. Wei, E. Fang, J. Lua. Integrating CAD with Abaqus: A Practical Isogeometric Analysis Software Platform for Industrial Applications. A Special Issue of HOFEIM 2016 in Computers and Mathematics with Applications, 74(7):1648-1660, 2017.
- [2] X. Wei, Y. J. Zhang, D. Toshniwal, H. Speleers, X. Li, C. Manni, J. Evans, T. J. R. Hughes. Blended B-Spline Construction on Unstructured Quadrilateral and Hexahedral Meshes with Optimal Convergence Rates in Isogeometric Analysis. Computer Methods in Applied Mechanics and Engineering, 341:609-639, 2018.
- [3] X. Wei, Y. J. Zhang, L. Liu, T. J. R. Hughes. Truncated T-splines: Fundamentals and Methods. Computer Methods in Applied Mechanics and Engineering Special Issue on Isogeometric Analysis, 316:349-372, 2017.

KN2 How to hear shape, style, and correspondence of 3D shapes - debunking some myths of spectral geometry

Mon, h. 10:20-11:20 in JRS 1420-1430

Chair: Carolina Beccari, University of Bologna, Italy.**Michael Bronstein**

Università della Svizzera Italiana, Switzerland,
Imperial College London, United Kingdom,
Intel Perceptual Computing, Israel.

**Biography**

Michael Bronstein is a professor at Imperial College London, where he holds the Chair in Machine Learning and Pattern Recognition, as well as a professor at the University of Lugano (USI), Switzerland. His main research expertise is in theoretical and computational methods for geometric data analysis, a field in which he has published extensively in the leading journals and conferences. He is credited as one of the pioneers in geometric deep learning, generalizing machine learning methods to manifold- and graph-structured data.

Michael received his PhD from the Technion (Israel Institute of Technology) in 2007. He has held visiting appointments at Stanford, MIT, Harvard, and Tel Aviv University. During 2017-2018 he was a fellow at the Radcliffe Institute for Advanced Study at Harvard University. Since 2017, he is a Rudolf Diesel fellow at the Institute for Advanced Study, TU Munich. Michael is the recipient of four ERC grants, two Google Faculty Research Awards, Royal Society Wolfson Merit Award, and Dalle Molle Foundation prize. He is a Fellow of IEEE and IAPR, ACM Distinguished Speaker, and World Economic Forum Young Scientist.

His industrial experience includes technological leadership in multiple startup companies, including Novafora, Invision (acquired by Intel in 2012), Videocites, and Fabula AI (acquired by Twitter in 2019). Following the acquisition of Fabula, Michael has joined Twitter as Head of Graph Learning Research. Previously, he has served as Principal Engineer at Intel Perceptual Computing (2012-2019).

Abstract

The question whether one can recover the shape of a geometric object from its Laplacian eigenvalues ('hear the shape of the drum') is a classical problem in spectral geometry with a broad range of implications and applications. While theoretically the answer to this question is negative (there exist examples of iso-spectral but non-isometric manifolds), little is known about the practical possibility of using the spectrum for shape reconstruction and optimization. In this talk, I will introduce a numerical procedure called isospectralization, consisting of deforming one shape to make its Laplacian spectrum match that of another. I will show the application of this procedure to some of the classical and notoriously hard problems in geometry processing, computer vision, and graphics such as shape reconstruction, pose and style transfer, and dense deformable correspondence.

In the second part of the talk, I will use perturbation analysis to show how removal of shape parts changes the Laplacian eigenfunctions, and exploit it for the construction of partial functional maps capable of addressing challenging real-world settings of deformable correspondence in the presence of clutter or occlusions.

KN3 Virtual Teleportation

Wed, h. 8:30-9:30 in Djavad Cinema

Chair: Richard Zhang, Simon Fraser University, Vancouver, Canada.**Shahram Izadi**
Google LLC, U.S.**Biography**

Dr. Shahram Izadi is a director at Google within the AR/VR division. Prior to Google he was CTO and co-founder of perceptiveIO, a Bay-Area startup specializing in real-time computer vision and machine learning techniques for AR/VR. His company was acquired by Alphabet/Google in 2017. Previously he was a partner and research manager at Microsoft Research (both Redmond US and Cambridge UK) for 11 years where he led the interactive 3D technologies (I3D) group. His research focuses on building new sensing technologies and systems for AR/VR. Typically, this meant developing new sensing hardware (depth cameras and imaging sensors) alongside practical computer-vision or machine-learning algorithms and techniques for these technologies. He was at Xerox PARC in 2000-2002, and obtained his PhD from the Mixed Reality Lab at the University of Nottingham, UK, in 2004. In 2009, he was named one of the TR35, an annual list published by MIT Technology Review magazine, naming the world's top 35 innovators under the age of 35. He has published over 120 research papers (see DBLP & Google Scholar), and more than 120 patents. His work has led to products and projects such as the Microsoft Touch Mouse, Kinect for Windows, Kinect Fusion, and most recently HoloLens and Holoportation.

Abstract

From the standpoint of the core technology, AR/VR has made massive advances in recent years, from consumer headsets to low-cost and precise head tracking. Arguably however, AR/VR is still a technology in need of the killer app. In this talk, I'll argue for why the killer app is immersive telepresence, aka virtual teleportation. The concept of virtual teleportation is not new, we've all been dreaming about it since the holograms of Star Wars. However, with the advent of consumer AR/VR headsets, it is now tantalisingly close to becoming fact rather than just science fiction. At its core, however, there's a fundamental machine perception problem still to solve – the digitization of humans in 3D and in real-time. In this talk I'll cover the work that we have done at Microsoft, perceptiveIO and now Google on this topic. I'll outline the challenges ahead of us and demonstrate some of the core algorithms and technologies that can get us closer to making virtual teleportation a reality in the future.



Minisymposia

MS1 Non-standard spline approximation schemes

Mon, h. 14:50-16:30 and 16:50-18:05 in JRS Centre 1420

Organizers: Cesare Bracco, University of Florence, Italy.
Carlotta Giannelli, University of Florence, Italy.

Standard spline representations rely on the univariate NURBS model and its tensor-product extension in the multivariate setting. Even if the regular structure of these geometries may simplify computations and construction of algorithms, its rigidity prevents the possibility of efficiently modeling and processing complex geometries and local features. The mini-symposium focuses on the most recent advances about non-standard spline spaces and their applications. The topics will include adaptive spline spaces, multi-patch spline constructions, splines on unstructured meshes, as well as subdivision schemes. In addition, related application algorithms in isogeometric analysis and image registration, as well as examples of industrial complexity, will be presented.

MS1.1 C^1 isogeometric spline spaces over planar multi-patch geometries

Mon, h. 14:50-15:15 in JRS Centre 1420

Mario Kapl¹, Johann Radon Institute for Applied and Computational Mathematics, Austrian Academy of Sciences, Austria.

Giancarlo Sangalli, Dipartimento di Matematica “F. Casorati”, Università degli Studi di Pavia, Italy.
Thomas Takacs, Institute of Applied Geometry, Johannes Kepler University, Linz, Austria.

Multi-patch spline parameterizations are a powerful tool in computer-aided geometric design to represent complex domains, and are used in isogeometric analysis to perform numerical simulations on these domains. This talk deals with the construction of C^1 isogeometric spline spaces over such planar multi-patch geometries, and demonstrates that these C^1 spline spaces are advantageous, since they allow the direct solution of fourth order partial differential equations on multi-patch geometries via their standard

Galerkin discretization.

In particular, we describe a method to generate a specific C^1 isogeometric spline space over a particular class of planar multi-patch spline parameterizations called analysis-suitable G^1 multi-patch parameterizations (cf. [1]). This class of parameterizations has to satisfy specific geometric continuity constraints, and is of importance since it allows to construct C^1 isogeometric multi-patch spaces with optimal approximation properties.

We first present an algorithm which approximates a given complex planar multi-patch domain by an analysis-suitable G^1 multi-patch parameterization [2]. Then, we use this multi-patch parameterization to construct the specific C^1 isogeometric spline space, which is a simpler subspace of the entire C^1 space maintaining the optimal approximation properties [3]. The design of this C^1 subspace is based on the construction of three different types of basis functions, which are locally supported and possess simple explicit representations. Finally, some numerical experiments are performed, which demonstrate the applicability of our approach for isogeometric analysis [4].

References:

- [1] A. Collin and G. Sangalli and T. Takacs. Analysis-suitable G^1 multi-patch parameterizations for C^1 isogeometric spaces, *Computer Aided Geometric Design*, 47, 93-113, 2016.
- [2] M. Kapl and G. Sangalli and T. Takacs. Construction of analysis-suitable G^1 planar multi-patch parameterizations, *Computer-Aided Design*, 97, 41-55, 2018.
- [3] M. Kapl and G. Sangalli and T. Takacs. An isogeometric C^1 subspace on unstructured multi-patch planar domains, *Computer Aided Geometric Design*, 69, 55-75, 2019.
- [4] M. Kapl and G. Sangalli and T. Takacs. Isogeometric analysis with C^1 functions on unstructured quadrilateral meshes, arXiv:1812.09088, 2018.

MS1.2 Quasi-interpolation quadratures for BEM with hierarchical B-splines

Mon, h. 15:15-15:40 in JRS Centre 1420

Tadej Kanduc[✉], INdAM, Italy.

In this talk I will present quadrature schemes for adaptive isogeometric boundary element method. The schemes combine a spline quasi-interpolation operator and analytical formulas for singular integrals. Local construction of the rules can readily be exploited when employing hierarchical B-spline shape functions and local refinement procedures. At the end I will show some Laplace model problems, where the optimal order of convergence for the approximate solution is recovered by the proposed adaptive model.

References:

- [1] Calabro F, Falini A, Sampoli ML, Sestini A. Efficient quadrature rules based in spline quasi-interpolation for application to IgA-BEMs. *J Comput Appl Math*. 2018;338:153–167.
- [2] Falini A, Giannelli C, Kanduc T, Sampoli ML, Sestini A. An adaptive IgA-BEM with hierarchical B- splines based on quasi-interpolation quadrature schemes. *Int J Numer Meth Eng*. 2019; 117:1038? 1058.
- [3] Falini A, Kanduc T. A study on spline quasi-interpolation based quadrature rules for the isogeometric Galerkin BEM. Accepted 2019. arXiv:1807.11277. Springer

MS1.3 Joint Image Segmentation and Registration Based on a Dynamic Level Set Approach Using Truncated Hierarchical B-splines

Mon, h. 15:40–16:05 in JRS Centre 1420

Aishwarya Pawar¹, Carnegie Mellon University, U.S.

Yongjie Jessica Zhang, Carnegie Mellon University, U.S.

Cosmin Anitescu, Bauhaus-Universität Weimar, Germany.

Timon Rabczuk, King Saud University, Riyadh, Saudi Arabia.

We present a novel approach for joint image segmentation and nonrigid registration using bidirectional composition-based level set formulation. This efficient framework incorporates automatic structural analysis from image segmentation into the registration framework. This method has shown an improved performance as compared to carrying out segmentation and registration separately. Unlike previous approaches, the implicit level set function defining the segmentation contour and the spatial transformation function that maps the deformation for image registration are both defined using B-splines. This joint level set framework uses a variational form of an atlas-based segmentation together with large deformation based nonrigid registration. In addition, a bidirectional composition framework is introduced to incorporate a more symmetric update. The minimization of the variational form is accomplished by dynamic evaluations on a set of successively refined adaptive grids at multiple image resolutions. The improvement in the description of the segmentation result using higher order splines lead to a better accuracy of both the image segmentation and registration process. The performance of the proposed method is demonstrated on synthetic and medical images to show the improvement as compared to other registration methods.

References:

- [1] A. Pawar, Y. J. Zhang, C. Anitescu, T. Rabczuk. Joint Image Segmentation and Registration Based on a Dynamic Level Set Approach Using Truncated Hierarchical B-Splines. *Computers and Mathematics with Applications*, 2019. DOI: 10.1016/j.camwa.2019.04.026

MS1.4 Industrial practices unlocked by adaptive splines

Mon, h. 16:05–16:30 in JRS Centre 1420

David Grossmann¹, MTU Aero Engines, Munich, Germany.

As one of the leading aircraft engine manufacturers, MTU Aero Engines is continuously extending the state-of-the-art in engineering design to increase its own long-term competitiveness. This requires highly efficient and flexible geometric technologies combining newest research results and real-world product design and repair inspection processes directly used by the specialists. Therefore, MTU invested heavily into the use of adaptive splines, i.e. truncated hierarchical B-splines (THB-splines) and patchwork B-splines (PB-splines), as generalizations of standard tensor-product splines, to unlock demanding industrial practices. The mini-symposium talk will give an overview of our latest results in using these promising concepts to extend the design space for an enhanced product performance and to improve the geometric re-construction for a tight CAD-CAE-CAM integration.

MS1.5 From CAD models to triangular spline surfaces

Mon, h. 16:50–17:15 in JRS Centre 1420

Jiri Kosinka¹, University of Groningen, Netherlands.

The standard representation of CAD (computer aided design) models is based on the boundary representation (aka B-reps) with trimmed and (topologically) stitched tensor-product NURBS patches. Due to trimming, this leads to gaps and overlaps in the models. While these can be made arbitrarily small for visualization and manufacturing purposes, they still pose problems in downstream applications such as analysis and simulation.

It is therefore worthwhile to investigate conversion methods which (necessarily approximately) convert these models into water-tight or even smooth representations. After briefly surveying existing conversion methods, we will focus on techniques that convert CAD models into triangular spline surfaces of various levels of continuity.

MS1.6 A Quasi-interpolation Method Based on LR B-splines

Mon, h. 17:15–17:40 in JRS Centre 1420

Francesco Patrizi¹, SINTEF, Norway, Carla Manni, University of Rome "Tor Vergata", Italy.
 Francesca Pelosi, University of Rome "Tor Vergata", Italy.
 Hendrik Speleers, University of Rome "Tor Vergata", Italy.

In order to break down the tensor structure of standard B-splines, Locally Refinable (LR) B-splines have been introduced in [1] by extending the concept of knot insertion of the 1D B-splines to local insertion of $(d-1)$ -dimensional boxes for the dD case. Like standard tensor-product B-splines, LR B-splines have local supports, are nonnegative and, using weights in $(0,1)$, they form a partition of unity. However, a full description of their linear independence is still an open problem. LR B-splines are defined over mesh instances, called LR-meshes. These are built as a sequence of local insertions starting from a coarse tensor mesh. In [2] there is provided a way of generating LR-meshes over which the corresponding LR B-splines are locally linearly independent. These meshes have a hierarchical structure, and the procedure requires an a priori knowledge of the subregions of the domain where the mesh should be finer. On the other hand, a quick and light construction of quasi-interpolation schemes based on Truncated Hierarchical (TH) B-splines has been developed in [3]. It is actually also applicable in the general setting where the basis functions have local supports, are nonnegative, form a partition of unity and are locally linearly independent. Moreover, it is proved that such a quasi-interpolant is actually a projector on the space spanned by the basis functions under some not-so-restrictive hypotheses on the given data set. In this talk we combine the above results to get a quasi-interpolant based on LR B-splines. We provide some numerical examples, and make comparisons with THB-splines and tensor-product B-splines.

References:

- [1] Tor Dokken, Tom Lyche, and Kjell Fredrik Pettersen. "Polynomial splines over locally refined box-partitions." *Computer Aided Geometric Design* 30.3 (2013): 331-356.
- [2] Andrea Bressan and Bert Jüttler. "A hierarchical construction of LR meshes in 2D." *Computer Aided Geometric Design* 37 (2015): 9-24.
- [3] Hendrik Speleers and Carla Manni. "Effortless quasi-interpolation in hierarchical spaces." *Numerische Mathematik* 132.1 (2016): 155-184.

MS1.7 Isogeometric methods with C^1 hierarchical spline spaces on planar two-patch geometries

Mon, h. 17:40–18:05 in JRS Centre 1420

Casare Bracco¹, University of Florence, Italy.

Carlotta Giannelli, University of Florence, Italy.

Mario Kapl, Johann Radon Institute for Computational and Applied Mathematics, Austria.

Rafael Vázquez, Ecole Polytechnique Fédérale de Lausanne, Switzerland.

Hierarchical splines are one of the most employed types of locally refinable spline spaces. In the context of Isogeometric Analysis (IgA) they have been used to construct adaptive methods for the numerical solution of PDEs with optimal convergence rates [1-3]. Since the isogeometric framework naturally offers the possibility to employ multipatch geometries and globally C^1 spline spaces defined on them to solve fourth-order PDEs, there is the need for spaces providing such continuity. Several works addressed this issue in the case of tensor-product spaces, with two or more patches [4-6]. In this talk, we will show that the combination of these techniques with the hierarchical framework allows us to obtain C^1 continuous hierarchical splines on two-patch domains. A selection of numerical examples will be presented.

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MS2 Volumetric parameterization and mesh generalization

Mon, h. 16:30–18:35 in JRS Centre 1430

Organizers: Falai Chen, University of Science and Technology of China, China

Xin Li, University of Science and Technology of China, China

Volumetric parameterization and mesh generalization is one of the most challenging problems in computer graphics, geometric modeling and computer aided engineering. They have been widely applied in texture mapping, shape morphing, spline construction, finite element method, and especially iso-geometric analysis (IGA)?an emerging technology which integrates computer aided design and computer aided engineering. While IGA has been successfully applied in various disciplines, such as structural mechanics, fluid structure interaction, phase-field modeling, etc., there are still several profound theoretical and practical problems that need further exploration. Among them,

volumetric parameterization and mesh generation are one of the facing challenges. The current mini-symposium aims to collect recent development in the mathematical foundations and algorithms of volumetric parameterization and mesh generation for IGA and/or geometric modeling.

MS2.1 Hexahedral Meshing and Holonomy

Mon, h. 16:30–16:55 in JRS Centre 1430

Na Lei, Dalian University of Technology, China.
 Feng Luo, Rutgers University, USA
 Jingyao Ke, University of Science and Technology of China, China.
 Xiaopeng Zheng, Dalian University of Technology, China.
 Zhongxuan Luo, Dalian University of Technology, China.
 Xianfeng Gu[✉], Stony Brook University, USA.

In this talk, we introduce the relation between a hexahedral mesh and holonomy group. Using algebraic topological method, one can construct a branched covering space of the hex-mesh, whose holonomy group is trivial. Furthermore, we analyze the topological structure of a 3-manifolds with a flat metric with singular curves, whose holonomy group is trivial. The special topological property leads to an effective computational algorithm.

MS2.2 Feature Preserving Hexahedral Meshing

Mon, h. 16:55–17:20 in JRS Centre 1430

Na Lei[✉], Dalian University of Technology, China.
 Xiaopeng Zheng, Dalian University of Technology, China.
 Zhongxuan Luo, Dalian University of Technology, China.
 Xianfeng Gu, Stony Brook University, USA.

Regular hexahedral meshing is crucial for IGA purposes. This work introduces a novel method for global regular hexahedral meshing, which preserves major sharp feature curves. The method is based on surface and volume foliations, which can be obtained by the metric graph valued harmonic maps. The feature curve preservation can be achieved by adding various types of constraints for the harmonic maps. The experimental results show the efficiency and efficacy of the proposed method.

MS2.3 IGA-suitable volume parameterization with geometric continuity along singular segmentation curves

Mon, h. 17:20–17:45 in JRS Centre 1430


Gang Xu[✉], Hangzhou Dianzi University, China.

In this paper, we propose a general framework for constructing IGA-suitable spline volume parameterizations from given complex CAD boundaries. Instead of the computational domain bounded by six B-spline surfaces, volumetric domains with high genus and more complex boundary surfaces are considered. Firstly, a robust volumetric domain partition framework is proposed to construct high-quality block-meshing results from the discrete boundary formed by connecting the corner points of the boundary segments. After the topology information generation of hexahedral decomposition, the optimal placement of interior segmentation surfaces corresponding to the interior faces of the hexahedralization is constructed by a global optimization method to achieve a block-partition with high quality. Finally, after the imposition of C^1/G^1 -continuity constraints on the interface of neighboring Bézier volumes with respect to each hex in

the hexhedralization, the high-quality Bézier volume parameterization is obtained by a local optimization method to achieve uniform and orthogonal iso-parametric structures while satisfying the geometric continuity conditions along singular segmentation curves. The efficiency and robustness of the proposed method are demonstrated by several examples with IGA applications.

MS2.4 Frame Field driven Hex Remeshing

Mon, h. 17:45–18:10 in JRS Centre 1430

Jin Huang , Zhejiang University, China.
Xianzhong Fang, Zhejiang University, China.

Hex mesh is attractive for many applications, but automatic and robust generation of high quality hex mesh is an open problem. Quadrangulation methods based on multi-chart parameterization achieve great success in these years. As a consequence, applying similar strategy for hex remeshing is a natural and promising idea. However, from the guiding field creation to the singularity structure analysis, many important steps in hex remeshing are much more difficult than the quad counterpart.

This talk will introduce our attempts about this problem. First, we proposed spherical harmonic representation[1] for general topology, analyzed and fixed all the local topological conflict[2]. Second, to bypass the challenge of global conflict, we aimed at the hex meshes without any internal singularity, started from the ones associated with exact form[3], and then extended the technique to the superset of exact form, i.e. closed form[4]. This talk will show our idea of building the nested topological structure from the general topology, closed form and exact form, and share some interesting findings of designing hex-meshable frame field.


References:

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MS2.5 Integrating design and nonlinear Kirchhoff-Love shell analysis using analysis-suitable unstructured T-splines

Mon, h. 18:10–18:35 in JRS Centre 1430

Hugo Casquero, Department of Mechanical Engineering, Carnegie Mellon University, U.S.
Xiaodong Wei, Institute of Mathematics, École Polytechnique Fédérale de Lausanne, Switzerland.
Deepesh Toshniwal, Institute for Computational Engineering and Sciences, The University of Texas at Austin, U.S.

Angran Li , Department of Mechanical Engineering, Carnegie Mellon University, U.S.
Thomas J.R. Hughes, Institute for Computational Engineering and Sciences, The University of Texas at Austin, U.S.

Josef Kiendl, Department of Marine Technology, Norwegian University of Science and Technology,

Norway.

Yongjie Jessica Zhang, Department of Mechanical Engineering, Carnegie Mellon University, U.S.

Analysis-suitable T-spline (ASTS) surfaces including both extraordinary points and T-junctions are used to solve Kirchhoff-Love shell problems. T-junctions enable to locally refine the region where increased resolution is needed [1]. Extraordinary points are required to represent geometries with arbitrary topology [2]. By using ASTS to define shell geometries, we can avoid the manual and time-consuming task of building a new mesh for engineering analysis using the CAD geometry as an input. In addition, we can apply the Galerkin method to discretize Kirchhoff-Love shells due to the C^1 or higher inter-element continuity of ASTS surfaces. The applicability of the proposed technology is illustrated by performing Kirchhoff-Love shell simulations of a pinched hemisphere, an oil sump, a T-shaped junction, and a B-pillar of a car [3]. Building ASTS surface meshes for these examples involves using T-junctions and extraordinary points with valences 3, 5, and 6, which often suffice for the design of free-form surfaces. We compare our analysis results with data from the literature using either Reissner-Mindlin shells or Kirchhoff-Love shells. We have also imported both finite element meshes and ASTS meshes into the commercial software LS-DYNA, used Reissner-Mindlin shells, and compared the result with our Kirchhoff-Love shell results. Excellent agreement is found in all cases.

References:

- [1] H. Casquero, L. Liu, Y. Zhang, A. Reali, J. Kiendl, H. Gomez, Arbitrary-degree T-splines for isogeometric analysis of fully nonlinear Kirchhoff-Love shells, *Computer-Aided Design*, 2017, 82, 140-153.
- [2] D. Toshniwal, H. Speleers, T. J. R. Hughes, Smooth cubic spline spaces on unstructured quadrilateral meshes with particular emphasis on extraordinary points: Geometric design and isogeometric analysis considerations, *Computer Methods in Applied Mechanics and Engineering*, 2017, 327, 411-458.
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MS3 Geometric Computing for CNC Machining

Tue, h. 10:20–11:35 in JRS Centre 1420

Organizer: Michael Barton, BCAM, Spain.

The MiniSymposium will be devoted to geometric modeling that is related to manufacturing, in particular to computer numerically controlled (CNC) machining. While design of curved objects is rather straightforward using a proper geometric modeling software, manufacturing of curved benchmark objects such as blades of turbines or parts of engines within very high machining precision introduces many difficult problems in the computer simulation stage. The MiniSymposium will discuss recent advances in modeling of various manufacturing technologies such as 3- and 5-axis CNC machining, hot-wire cutting, and will also focus on validation algorithms in the post-manufacturing stage (quality inspection).

MS3.1 Design for hot-blade cutting

Tue, h. 10:20–10:45 in JRS Centre 1420

Ann-Sofie Fisker, Department of Applied Mathematics and Computer Science, DTU, Denmark.
David Brander, Department of Applied Mathematics and Computer Science, DTU, Denmark.

Andreas Bærentzen, Department of Applied Mathematics and Computer Science, DTU, Denmark.
 Jens Gravesen[✉], Department of Applied Mathematics and Computer Science, DTU, Denmark.

Euler's elastica are natural curves that appeared in design before the introduction of computers, because these curves are the natural shape for the wooden splines that were used in drafting. Although they have natural aesthetic properties, they were abandoned when computers were introduced because polynomial splines are much easier to deal with computationally. Motivated by hot-blade cutting (a new fabrication processes), we have studied the practical problem of designing with elastic curves and splines on a computer. We show in [1] that the problems of non-uniqueness and instability, inherent in the nonlinear mathematics of elastic splines, can be solved by introducing a suitable proxy curve to represent the elastica. In this talk, I will describe this work, as well as some remaining issues.

1. Bézier curves that are close to elastica. D. Brander, J.A. Bærentzen, A. Fisker and J. Gravesen, *Computer-Aided Design*, 104, 36-44, (2018). DOI: 10.1016/j.cad.2018.05.003.
2. Approximation by planar elastic curves. D. Brander, J. Gravesen and T. Nørbjerg. *Adv. Comput. Math.* 43, 25-43, (2017). DOI: 10.1007/s10444-016-9474-z

MS3.2 A Novel Robotic Multi-axis Additive Manufacturing System

Tue, h. 10:45–11:10 in JRS Centre 1420

Li Chen[✉], The Hong Kong University of Science and Technology, Hong Kong.
 Kai Tang, The Hong Kong University of Science and Technology, Hong Kong.

The current additive manufacturing (AM) technologies still suffer from numerous setbacks, e.g. low surface quality, excessive supporting structure, and poor mechanical performance, most of which can be attributed to the nature of the 2.5-axis setup of the current AM system. In this work, we built a novel hybrid robotic AM system with the capability of multi-directional printing and curved-layer printing, which is able to minimize the staircase effect, eliminate support structure, and enhance the mechanical performance of the workpiece. In the current iteration our AM system, a 6-DOF robotic arm is utilized on which the heating bed is mounted, while the extruder is fixed on a 2-DOF XY-table. Free simultaneous movement between all axes (include extrusion) is achieved. Aside from platform building, post-processing and motion control. Multi-axis additive path planning still faces many of its own unique challenges e.g. collision problem, adherence problem, and non-uniform thickness deposition problem. In this work, we introduced a computational framework consists of primitive based workpiece decomposition, curved layer construction, optimal step-over layer path planning and collision-free printing process planning that effectively and robustly generates high-quality and support-free path/process of multi-axis additive manufacturing of complex parts. Finally, ample test examples are given to demonstrate the validity of our methodology.

MS3.3 Highly-accurate 5-axis flank CNC machining with conical tools

Tue, h. 11:10–11:35 in JRS Centre 1420

Amaia Calleja, the University of the Basque Country, Spain.
 Pengbo Bo, School of Computer Science and Technology, Harbin Institute of Technology, China.
 Haizea González, Department of Mechanical Engineering, the University of the Basque Country, Spain.
 Michael Bartoň[✉], BCAM – Basque Center for Applied Mathematics, Spain.

Luis Norberto López de Lacalle, Department of Mechanical Engineering, the University of the Basque Country, Spain.

A new method for 5-axis flank computer numerically controlled (CNC) machining using a predefined set of tapered ball-end-mill tools (aka conical) cutters is proposed. The space of lines that admit tangential motion of an associated truncated cone along a general, doubly curved, free-form surface is explored. These lines serve as discrete positions of conical axes in 3D space. Spline surface fitting is used to generate a ruled surface that represents a single continuous sweep of a rigid conical milling tool. An optimization based approach is then applied to globally minimize the error between the design surface and the conical envelope. Our computer simulation are validated with physical experiments on two benchmark industrial datasets, reducing significantly the execution times while preserving or even reducing the milling error when compared to the state-of-the-art industrial software.

MS4 Industrial Strength Geometric Modeling

Tue, h. 10:20–12:00 in JRS Centre 1430

Organizers: Thomas Grandine, The Boeing Company, U.S.
Huseyin Erdim, The Boeing Company, U.S.

Performing geometric modeling in an industrial setting creates new and difficult challenges beyond the usual mathematical ones. This minisymposium will explore some of these additional challenges which include long term model retention, robustness of geometric operations, and effective modeling for additive manufacturing and 3D printing.

MS4.1 Hybridization of Manufacturing: Challenges and Future Directions

Tue, h. 10:20–10:45 in JRS Centre 1430

Morad Behandish¹, Palo Alto Research Center (PARC), U.S.
Saigopal Nelaturi, Palo Alto Research Center (PARC), U.S..

Additive manufacturing (AM) remains in the spotlight of academic research, partly due to its democratization of making and ability to produce complex shapes and graded materials. AM has also shown great promise for industry-strength fabrication. However, it is clear that in foreseeable future, AM will not completely replace subtractive manufacturing (SM) such as high-precision machining of assembly interfaces. SM remains central to making industrial and mission-critical parts because of its precision, quality, reliability, and (in some cases) lower cost. The near future of parts manufacturing will be hybrid. Hybrid manufacturing (HM) machines incorporating both AM and SM enable seamless switching between material addition and removal. Design for hybrid manufacturing (DfHM) is a next frontier for important and exciting industry-strength geometric modeling research, and there are substantial challenges. Existing tools on unimodal (AM-only or SM-only) process planning cannot be readily extended to multi-modal HM due to the absence of monotonicity or permutativity of AM/SM operations with one another. Limited scalability and interoperability of process planning tools is of prime importance in industrial workflows, despite receiving less attention than deserved in academic research. Incorporating HM planning within industrial workflows with minimal disruption is key to adoption. We present an approach to HM planning that decouples geometric and physical reasoning from planning (using

AI search) in a way that the latter can be completely re-stated in symbolic/logical terms. We also discuss an HM "meta-planner" that orchestrates AM/SM services, given handles to simple queries that most geometric kernels already support. If the services are pairwise interoperable (at least supporting pairwise intersection queries), the meta-planner delegates all geometric operations. We show how this abstraction enables process planning as a service, easily incorporated within or interoperating with existing CAD/CAM systems.

MS4.2 A Language for Ultra-Long-Term CAD Data Preservation

Tue, h. 10:45–11:10 in JRS Centre 1430

Walter Wilson¹, Lockheed Martin, U.S.

Computer-aided design data should ideally last the lifetime of the artifact it describes. For an aircraft program that could be decades. For buildings, centuries, maybe a thousand years. This talk will give some principles for "ultra-long-term" CAD data preservation. (www.axiomaticlanguage.org/LOTAR_Thoughts.html) A type of logic programming called "axiomatic language" will be proposed as an ideal foundation for representing engineering design data. (www.axiomaticlanguage.org/A_Vision_for_CAD_released.pdf)

MS4.3 Rolling up our SLEFEs: Reflections on Putting an Academic Rendering Algorithm to Work

Tue, h. 11:10–11:35 in JRS Centre 1430

Craig Bosma¹, The Boeing Company, USA.

At the Boeing Company, transitioning an algorithm presented in a paper or pre-production code to a robust, full-fledged component of a production system often presents challenges not typically addressed by academic research. In this talk, we present some of these unique challenges encountered while implementing a new visualization capability for GEODUCK, a Boeing-developed geometric design tool. In particular, we will briefly review SLEFEs (Subdividable Linear Efficient Function Enclosures) [Peters, et.al.] for accurate rendering of nonlinear geometry on GPUs, our implementation of the SLEFE algorithm, and some of the difficulties associated with deploying a GPU algorithm in a diverse hardware/software landscape. We will also cover how off-the-shelf tools and techniques for debugging and profiling of GPU algorithms were employed to improve the robustness and performance of the production-ready software. In addition, we will speak to how certain well-known scientific visualization techniques were integrated into our system. In conclusion, we will present our goals for further improvements to this visualization capability and opportunities for future research that will enable the next iteration of accurate, interactive scientific and engineering visualization.

MS4.4 Geometric design challenges for additive manufacturing

Tue, h. 11:35–12:00 in

JRS Centre 1430

Huseyin Erdim¹, The Boeing company, U.S.

Thomas Grandine, The Boeing company, U.S.

Full featured geometric design tools for additive manufacturing, especially with laminated composite materials, are still in their infancy. The tools needed for these capabilities involve complex calculations and geometry processing operations on both geometric and solid models. The lack of high quality tools impedes exploration of design and manufacturing alternatives to improve manufacturing throughput and reduce

cost. This talk will discuss some of the computational methods developed to support laminated composite fabrication as well as more conventional 3D printing.

MS5 Topology, Geometry and Graphics for Design – Applications & Theory

Tue, h. 13:30–15:10 in JRS Centre 1420

Organizer: Thomas Peters, University of Connecticut, U.S.

Design relies upon the traditional mathematics of topology and geometry, with significant insights gained from advancements in computer graphics. The richness of applications (from molecules to airplanes) and emerging theory for future advances will be presented.

MS5.1 Topology & Convexity for Optimization in Chemical Engineering

Tue, h. 13:30–13:55 in JRS Centre 1420

Thomas Peters , University of Connecticut, U.S.

Molecular simulations produce data sets for analyses to optimize chemical engineering designs of micelles. These chemicals have applications in detergents, shampoo, oil extraction and food preservation. Geometric shape can determine chemical properties. Each molecule is represented by a cluster of (x, y, z) points. An initial topological exercise is to extract a boundary, leading to convexity algorithms to categorize each shape as approximately spheroidal, ellipsoidal, cylindrical or worm-like (similar to a bent garden hose). Future work will rely on topological surgery to join these basic building blocks.

MS5.2 Determining Color Location Properties With Respect to a Color Gamut

Tue, h. 13:55–14:20 in JRS Centre 1420

Dan Gonsor , The Boeing Company, U.S.

In this talk we give results related to determining various color location properties of a specified color with respect to a color gamut. In particular, we investigate containment classification, and distance from color gamut with respect to an arbitrary distance function. Such results have applications in devices producing color matches from a limited number of replications.

MS5.3 Clifford Algebra Representations for Computer Graphics

Tue, h. 14:20–14:45 in JRS Centre 1420

Stephen Mann , University of Waterloo, Canada.

Computer graphics takes an ad hoc, piecemeal approach to mathematics; while everything works, one has to be careful to avoid the places that this ad hoc mathematics fails and gives incorrect results. Several Clifford Algebras have been suggested for use in computer graphics. These models all provide a solid mathematical basis for the operations and objects used in computer graphics. In this talk, I will review models, touching on how they are applicable to surface modeling.

MS5.4 Mathematical Abstractions for Engineering Design and Manufacturing

Tue, h. 14:45–15:10 in JRS Centre 1420

Jiangce Chen, University of Connecticut, U.S.
Morad Behandish, PARC, U.S.
Horea Ilies[✉], University of Connecticut., U.S.

NURBS based boundary representations are the de-facto standard in most if not all commercial CAD systems. One of their key advantages is that they provide an intuitive and accurate mechanism for creating geometric models in terms of geometrically meaningful control points, but they do not offer appropriate computational support for important downstream applications. In this talk we discuss some of our recent efforts in adapting or redefining geometric representations, such as those based on overlapping or disjoint spherical decompositions, to support specific classes of applications including shape similarity/segmentation as well as interactive haptic assembly of complex shapes.

MS6 Advanced Shape Optimization: Non-smoothness and Time-Dependency

Tue, h. 13:30–15:10 in JRS Centre 1430

Organizers: Stephan Schmidt, University of Würzburg, Germany
Roland Herzog, Technical University Chemnitz, Germany

The aim of this mini-symposium is mathematical shape optimization with a special focus on time-dependent and non-smooth geometries.

Such problems naturally arise in geometric inverse problems, e.g., the reconstruction of non-smooth moving objects from measurements. As such, we will on the one hand discuss geometric regularization terms that foster the creation of non-smooth objects such as the total variation of the normal. On the other hand, we will also focus on tube-derivatives and the efficient computation of sensitivities of integrals with respect to morphing domains, which typically results in a variant of the tube derivative.

Special attention will in particular also be given on the efficient computation and realization of the above ideas on triangulated domains and surfaces. This results in questions of appropriate finite element or space-time finite element spaces on surfaces.

MS6.1 Multi-Mesh Shape Optimization and Higher Shape Derivatives

Tue, h. 13:30–13:55 in JRS Centre 1430

Stephan Schmidt[✉], University of Würzburg, Germany.
Jørgen Schartum Dokken, Simula Research Laboratory, Norway.
Simon Funke, Simula Research Laboratory, Norway.

The aim of the talk is to discuss shape optimization for translation and rotation dominant problems. Such problems typically arise in shape optimization of rotary objects or in the optimization of the positioning in ensembles. Typical applications are tidal turbine placement optimization and wind park placings.

Those difficulties can be overcome by a multi mesh FEM approach and the main focus of the presentation is shape optimization on multiple meshes. Special attention is given to the best choice of boundary- and/or surface formulations. Higher Shape derivatives will be discussed, too. The presentation concludes with a shape-Newton method for CFD.

MS6.2 Geometric Inverse Problems and the Total Variation of the Normal

Tue, h. 13:55–14:20 in JRS Centre 1430

Marc Herrmann¹, Universität Wuerzburg, Germany.

Roland Herzog, TU Chemnitz, Germany.

Jose Vidal, Technische Universität Chemnitz, Germany.

Stephan Schmidt, University of Würzburg, Germany.

Ronny Bergmann, Technische Universität Chemnitz, Germany.

An analogue of the total-variation prior for the normal vector field along the boundary of smooth shapes in 3D is introduced. The novel functional turns out to agree with the total root mean square curvature. This notion is subsequently extended to piecewise flat, triangulated surfaces as they occur for instance in finite element computations. A split Bregman iteration is proposed for the solution of shape optimization problems in which the total variation of the normal appears as a regularizer. In numerical practice, the discrete functional allows for piecewise flat, convex shapes. As an application, an inclusion detection problem is considered, in which convex polyhedral shapes can be identified in the noise-free case.

MS6.3 On the solution of a time-dependent inverse shape identification problem for the heat equation

Tue, h. 14:20–14:45 in JRS Centre 1430

Rahel Brügger¹, University of Basel, Switzerland.

Helmut Harbrecht, University of Basel, Switzerland.

Johannes Tausch, Southern Methodist University, U.S.

In the talk we treat the solution of a time-dependent shape identification problem. We specifically consider the heat equation on a domain, which contains a star-shaped inclusion of zero temperature. We aim at detecting this time-dependent inclusion by measuring the heat flux on the exterior boundary of the domain. Reformulation by using a Neumann data tracking functional leads to a time-dependent shape optimization problem, for which a gradient based method is considered.

MS6.4 Total variation of the normal: properties, discretization and variational problems

Tue, h. 14:45–15:10 in JRS Centre 1430

Roland Herzog¹, TU Chemnitz, Germany.

Stephan Schmidt, University of Würzburg, Germany.

Marc Herrmann, University of Würzburg, Germany.

Ronny Bergmann, Technische Universität Chemnitz, Germany.

Jose Vidal-Nunez, Technische Universität Chemnitz, Germany.

The total variation (TV) is an important regularizing seminorm in inverse problems and in particular in imaging applications. In this presentation we consider problems where the shape is among the unknowns. We define the notion of total variation of the surface normal as a prior for this class of problems and discuss this term in the continuous and discrete settings on triangulated surfaces. We also address properties of minimizers as well as a suitable numerical scheme to deal with the non-smoothness arising from the TV of the normal.

MS7 Generalized Barycentric Coordinates in Computational Mechanics

Tue, h. 15:30–17:10 in JRS Centre 1420

Organizer: N. Sukumar, University of California, Davis, U.S.

Over the past decade, generalized barycentric coordinates on polygons and polyhedra have had broad applications in computer graphics and computational mechanics. As an alternative to traditional Galerkin finite element methods on simplices, the use of generalized barycentric coordinates on polytopal meshes has provided new opportunities for mathematical analysis as well as Galerkin formulations for the modeling and simulation of physical phenomena. The contributed talks in this minisymposium showcase some of the recent advances and challenges in this emerging area, such as polyhedral finite element mesh generation and its use towards a design-to-analysis pipeline, construction of bivariate spline spaces on polygons, applications of polyhedral discretization in nonlinear solid mechanics, and recent advances in the numerical integration of homogeneous functions over domains with curved boundaries.

MS7.1 Polyhedral discretizations using tetrahedral subdivisions, aggregation, and optimization-based shape functions for applications in nonlinear solid mechanics

Tue, h. 15:30–15:55 in JRS Centre 1420

Joseph Bishop¹, Sandia National Laboratories, U.S.

For geometrically complex parts and systems, the time required to develop an analysis capable finite element mesh can still take days to months to develop, despite vast increases in computing power. Advanced tetrahedral meshing tools can alleviate this burden, but the development of robust and efficient tetrahedral finite-element formulations for applications in large-deformation solid-mechanics is still an active area of research. The recent development of general polyhedral formulations for solid mechanics offers an opportunity to ease this meshing burden.

Starting with an existing tetrahedral mesh, we first subdivide each tetrahedron using standard rectification techniques. Two types of polyhedra are formed from the tetrahedral subdivisions: (1) an aggregation of subtetrahedra attached to the original nodes, and (2) either an octahedron with 6 vertices or a polyhedron with 12 vertices and 8 faces, depending on the degree of rectification. Several approaches may then be used to define the shape functions (e.g. using harmonic or maximum entropy coordinates) and quadrature schemes for the new polyhedra to obtain a consistent and stable finite element formulation. Verification problems and several large-deformation examples are presented.

MS7.2 First Steps Toward Black-Box Finite Element Analysis

Tue, h. 15:55–16:20 in JRS Centre 1420

Teseo Schneider¹, NYU Courant Institute of Mathematical Sciences, U.S.

The numerical solution of partial differential equations (PDE) is ubiquitous in computer graphics and engineering applications, ranging from the computation of UV maps and skinning weights, to the simulation of elastic deformations, fluids, and light scattering. Ideally, a PDE solver should be a "black box": the user provides as input the domain boundary, boundary conditions, and the governing equations, and the code returns an evaluator that can compute the value of the solution at any point of the input domain. This is surprisingly far from being the case for all existing open-source or commercial software, despite the research efforts in this direction and the large academic and industrial interest. To a large extent, this is due to treating meshing and FEM basis construction as two disjoint problems.

We will present an integrated pipeline, considering meshing and element design as a single challenge, that makes the tradeoff between mesh quality and element

complexity/cost local, instead of making an a priori decision for the whole pipeline. We will demonstrate that tackling the two problems jointly offers many advantages, and that a fully black-box meshing and analysis solution is already possible for heat transfer and elasticity problems.

This is a joint effort of Teseo Schneider, Yixin Hu, Jeremie Dumas, and Xifeng Gao, Daniele Panozzo, Denis Zorin.

MS7.3 A Minimization Approach for Constructing Generalized Barycentric Coordinates

Tue, h. 16:20–16:45 in JRS Centre 1420

Ming-Jun Lai[✉], University of Georgia, U.S.

We are interested in constructing more generalized barycentric coordinates (GBC) over arbitrary polygon in 2D and 3D settings. We propose a constrained minimization over the class of infinitely differentiable functions subject to the GBC constraints of preserving linear functions and the non-negativity condition. It includes the harmonic GBC, biharmonic GBC, maximum entropy GBC, local barycentric coordinates as special cases. We mainly show that the constrained minimization has a unique solution when the minimizing functional is strictly convex. Next we use a C^r smoothness spline function space $S_d^r(\Delta)$ with $r \geq 2$ over a triangulation Δ of any polygon in \mathbb{R}^2 to approximate the minimizer. The minimization restricted to the spline space S_d^r certainly has a unique minimizer. Then we propose to use the standard projected gradient descent (PGD) method to approximate the minimizer. To find the projection of the gradients, we shall explain how to use an alternating projection algorithm (APA). A convergence of the APA and the convergence of the PGD with the APA will be presented. Based on this approach, a new kind of biharmonic GBC functions which preserve the nonnegativity are defined. Finally, we have implemented the PGD method based on bivariate splines. The surfaces of many new GBC's will be shown. Some standard GBC applications will be demonstrated. This talk is based on a joint work with Chongyang Deng and Xiali Fang.

MS7.4 Integration of Homogeneous Functions on Curved Geometries

Tue, h. 16:45–17:10 in JRS Centre 1420

Eric B. Chin, University of California, Davis, U.S.

N. Sukumar[✉], University of California, Davis.

Numerical integration over curved geometries has become a growing need in many emerging computational methods such as the extended finite element method, embedded interface methods, and polyhedral approaches such as the virtual element method. In this talk, an extension of the homogeneous numerical integration (HNI) method [1] to two-dimensional domains bounded by parametric curves (rational Bezier curves and cubic Hermite splines) will be presented. The HNI method combines Euler's homogeneous function theorem and Stokes's theorem to reduce integration to the boundary of the geometry. In [1], regions of integration are limited to those bounded by affine edges, such as convex and nonconvex polygons, and by homogeneous algebraic curves, such as circular arcs. In this work, we extend the approach to regions bounded by arbitrary parameterized curves. This provides a means to efficiently and accurately integrate positively homogeneous functions on such geometries. Further, if the curved boundary is defined as a polynomial curve and the integrand is polynomial, the HNI method delivers exact integration with an appropriate Gauss quadrature rule. For curved domains too, the HNI approach eliminates the need for partitioning the

domain; it also reduces the dimensionality of integration, which considerably reduces the number of cubature points required for accurate integration. Illustrative numerical examples will be presented to highlight the capabilities of the HNI method for integrating over curved geometries.

References:

- [1] Chin EB, Lasserre JB, Sukumar N. Numerical integration of homogeneous functions on convex and nonconvex polygons and polyhedra. *Comput Mech* 2015; 56(6): 967-981.

MS8 V-rep and IgA for Design and Additive Manufacturing

Tue, h. 16:30–18:35 in JRS Centre 1430

Organizers: Tor Dokken, SINTEF, Norway
Gershion Elber, Technion, Israel

The emerging V-rep (Volumetric representation) technology for shape representation is based on structures of trimmed tri-variate spline blocks. The representation is well suited for the representation of variable/graded materials and anisotropic material properties integral in AM processes. V-rep is fundamentally different from traditional CAD-technologies, which only represent the boundaries of an object (B-rep). In B-rep CAD, it is assumed that the material is uniform all throughout an object, so only the surfaces must be represented. In Isogeometric Analysis (IgA), traditional shape functions used in Finite Element Analysis (FEA) are replaced by B-splines. Thus, V-rep opens for general simulations in AM, based on the accurate shape representation of V-rep based CAD, and the use of higher order elements offered by IgA. This allows the execution of design and analysis operations on the same geometric representation. The mini-symposium aims at addressing solutions and challenges related to the use of V-rep and IGA for solving design and AM-challenges, including in the context of heterogeneous materials and porous/microstructures geometry.

MS8.1 V-rep and IgA for simulation based design for Additive Manufacturing in the CxMan-project

Tue, h. 16:30–16:55 in JRS Centre 1430

Tor Dokken¹, SINTEF Digital, Norway.

Additive Manufacturing (AM) is a collective term for processes that join material, usually layer upon layer, to make parts from 3D model data. AM creates parts with new, variable and anisotropic material properties, innovative outer shape and advanced inner structures not possible to produce with subtractive and formative technologies. Traditional CAD-representation (B-rep) assumes uniform material in the interior of parts and represents the surfaces of an object. In 2018 new tri-variate spline representations were introduced in ISO 10303 (STEP) to support IGA, variable material and the advances shapes allowed by AM. There is an ongoing process to add topological structures to STEP (V-rep) that compose trimmed trivariate spline volumes into larger objects. The presentation will focus on how V-rep and IGA were used in the EC H2020 project www.caxman.eu (2015-2018) to address simulation-based design for AM.

MS8.2 Designing Heterogeneous Materials/Microstructures

Tue, h. 16:55–17:20 in JRS Centre 1430

Gershon Elber[✉], Technion– Israel Institute of Technology, Israel.

The needs of modern (additive) manufacturing (AM) technologies can be satisfied no longer by boundary representations (B-reps), as AM requires the representation and manipulation of interior fields and materials as well. Further, while the need for a tight coupling between design and analysis has been recognized as crucial almost since geometric modeling (GM) has been conceived, contemporary GM systems only offer a loose link between the two, if at all.

For about half a century, (trimmed) Non Uniform Rational B-spline (NURBs) surfaces has been the B-rep of choice for virtually all the GM industry. Fundamentally, B-rep GM has evolved little during this period. In this talk, we seek to examine an extended volumetric representation (V-rep) that successfully confronts the existing and anticipated design, analysis, optimization, and manufacturing foreseen challenges. We extend all fundamental B-rep GM operations, such as primitive and surface constructors and Boolean operations, to trimmed trivariate V-reps. This enables the much needed tight link to (Isogeometric) analysis on one hand and the full support of (heterogeneous and anisotropic) additive manufacturing on the other.

Further, special capabilities in supporting Isogeometric analysis will also be presented, that enable robust queries over the V-reps, including precise contact analysis, maximal penetration depth, and accurate integration over trimmed domains. Examples and other applications of V-rep GM, including AM and lattice- and micro-structure synthesis (with heterogeneous materials) will also be demonstrated.

In collaboration with many others, including Ben Ezair, Fady Massarwi, Boris van Sosin, Jinesh Machchhar, Annalisa Buffa, Giancarlo Sangalli, Pablo Antolin, Massimiliano Martinelli, Bob Haimes, and Stefanie Elgeti.

MS8.3 Numerical methods on V-reps.

Tue, h. 17:20–17:45 in JRS Centre 1430

Pablo Antolin, Ecole Polytechnique Fédérale de Lausanne, Switzerland.

Annalisa Buffa[✉], Ecole Polytechnique Fédérale de Lausanne, Switzerland.

Riccardo Puppi, Ecole Polytechnique Fédérale de Lausanne, Switzerland.

Rafael Vazquez, Ecole Polytechnique Fédérale de Lausanne, Switzerland.

Following the new paradigm of V-reps (volumetric representation) introduced by Massarawi et al in 2016, we move the first steps towards the design of robust numerical methods for PDEs on V-reps. Several challenges are to be tackled in order to achieve the goal of having a versatile, robust, and accurate PDE solver on V-reps, able to simulate several different physics, from diffusion to structural mechanics or fluids. In this talk, we will discuss the design of numerical methods for diffusion and (linear) elasticity on V-reps, both from the theoretical and algorithmic point of view.

MS8.4 SimuLearn: Combining Finite Element Simulation and Machine Learning for Inverse Design and Manufacture of Self-Assembling Viscoelastic Materials

Tue, h. 17:45–18:10 in JRS Centre 1430

Yuxuan Yu[✉], Computational Bio-modeling Laboratory, Department of Mechanical Engineering, Carnegie Mellon University, U.S.

Humphrey Yang, Morphing Matter Lab, Human-Computer Interaction Institute, School of Computer Science, Carnegie Mellon University, U.S.

Haolin Liu, Morphing Matter Lab, Human-Computer Interaction Institute, School of Computer

Science, Carnegie Mellon University, U.S.

Kuanren Qian, Computational Bio-modeling Laboratory, Department of Mechanical Engineering, Carnegie Mellon University, U.S.

Jianzhe Gu, Morphing Matter Lab, Human-Computer Interaction Institute, School of Computer Science, Carnegie Mellon University, U.S.

Lining Yao, Morphing Matter Lab, Human-Computer Interaction Institute, School of Computer Science, Carnegie Mellon University, U.S.

Yongjie Zhang, Computational Bio-modeling Laboratory, Department of Mechanical Engineering, Carnegie Mellon University.

4D printing is a technology that leverages additive manufacturing to program post-fabrication deformation into objects. This fabrication approach allows us to fabricate structures that are difficult to produce with conventional 3D printing, such as non-developable structures. Previously, our team leveraged 4D printing to produce polylactic acid (PLA) artifacts that are initially fused deposition modeling (FDM) printed as flat pieces and can later be triggered to transform into their target shapes on demand. This transformation is driven by releasing the residual stresses in the PLA induced during the printing process. However, the workflow lacks an effective simulation tool that provides fast and truthful previews of the transformation process, hence making design iterations difficult and the transformation accuracy difficult to control. To address this challenge, we propose a finite element analysis (FEA)-based data-driven simulation technology that leverages machine learning (ML). This technology provides designers and engineers with fast (compared to FEA) and faithful simulations (compared to purely geometrical modeling) for beam- and mesh-like PLA structures. Nonetheless, this method is adaptable to various materials and structures. Firstly, to compose a simulation engine for the aforementioned structures, we conduct several experiments to characterize the deformation behaviors and material properties of PLA at 800C, slightly above its glass transition temperature. These tests include uniaxial tensile and compression tests for the elastic component and dynamic mechanical analysis for the viscous component at the elevated temperature. We then generate FE simulations for beam- and grid-topological patterns with the collected properties of PLA to collect data for training an ML model. Lastly, the trained ML model as a simulation engine is capable of simulating the transformation of beam- or mesh-like PLA structures with speed and accuracy. Our current implementation shows an 3000x increase in speed with a 5% average error rate compared to the reference FEA results. With this ML-based FEA method, users can preview the transformation and iterate on the design in real-time to ensure higher transformation accuracies and explore design alternatives.

MS8.5 Design and Fabrication at the Edge of Natural Systems

Tue, h. 18:10–18:35 in JRS Centre 1430

Turlif Vilbrandt¹, Uform, Norway.

3D scanning and data collection from the physical world, complex multi-physics simulation, additive manufacturing, and bio-engineering are allowing us to construct objects of unprecedented complexity and quality reflective of natural systems. However, traditional approaches to design, computational frameworks, and software tools are insufficient to realize the full potential of these capabilities. New multi-dimensional volumetric technologies are being invented and used to push human design to the edge of nature, 'grow' objects rather than design them, and digitally imbue them with properties, materials and behavior. An overview of the serious issues and problems

facing digital design tools and processes is outlined along with a brief survey of volumetric methods and techniques that can provide solutions to these problems. Volume modeling use cases already in real world practice today are illustrated.

MS9 Design of rational rigid body motions


Wed, h. 16:20–18:00 in JRS Centre 1420

Organizer: Rida Farouki, University of California, Davis, U.S.

The design of a spatial motion involves specifying position and orientation at each instant. The path of a distinguished point (e.g., the center of mass) may be used to specify position as a space curve, and an orthonormal frame defined along this curve may be used to specify orientation. In recent years, considerable progress has been made in constructing motions specified by rational curves and rational orthonormal frames, including the interpolation of discrete positions and orientations, and the construction of motions characterized by correlations of position and orientation through various frame types (including rotation-minimizing, minimal twist, directed, and osculating frames). Such spatial motions have diverse applications in computer animation, robotics, 5-axis CNC machining, swept surface constructions, and related fields.

MS9.1 Rational orthonormal frames along space curves


Wed, h. 16:20–16:45 in JRS Centre 1420

Rida Farouki , University of California, Davis, U.S.

A constrained spatial rigid-body motion involves correlation of position and orientation. The definition and properties of several recently-developed rational orthonormal frames, that can be used to specify such constrained spatial motions, are surveyed. A rotation-minimizing frame (RMF) maintains a zero angular velocity component in the path tangent direction. Since the construction of an RMF corresponds to an initial-value problem, it cannot be employed to interpolate prescribed orientations at two distinct points. To address this boundary-value problem, the minimal twist frame (MTF) is introduced. The MTF exhibits the least total rotation about the tangent, of consistent sense, required to satisfy the boundary conditions. Frames that are rotation-minimizing with respect to the curve principal normal, binormal, and polar vector are also discussed, and several applications of these different constrained frames are highlighted.

MS9.2 Motion interpolation using Pythagorean-hodograph curves with prescribed length

Wed, h. 16:45–17:10 in JRS Centre 1420

Marjeta Knez , IMFM and Faculty of Mathematics and Physics, University of Ljubljana, Slovenia.

Polynomial Pythagorean-hodograph (PH) curves in space, which are characterized by the property that the unit tangent is rational, have many important features for practical applications. One of them is that these curves can be equipped with rational orthonormal frames called Euler-Rodrigues (ER) frames, where the first frame vector coincides with the unit tangent. The second important property is that the arc-length function is a polynomial. Joining these two properties we can construct motions of a rigid-body that interpolate some given positions and have a prescribed length of the center trajectory. In the talk, the G^1 interpolation of motion data, i.e., interpolation of

data points and rotations at the points, with PH curves is presented, where the rotational part of the motion is determined by the ER frame. In addition, the length of the center trajectory is prescribed. It is shown how to construct the interpolants using cubic PH biarc curves and quintic PH curves. In both cases the solutions exist for any data and any length greater than the difference between the interpolation points. Moreover, the interpolants depend on some free parameters, which can be chosen so that the center trajectory is of a nice shape and the twist of the ER frame is minimized. The derived theoretical results are illustrated with numerical examples. Possible extensions to PH curves of higher degree and higher order of smoothness are suggested.

MS9.3 Construction of rational adapted frames along closed spatial Pythagorean hodograph curves

Wed, h. 17:10–17:35 in JRS Centre 1420

Hwan Pyo Moon[✉], Dongguk University-Seoul, South Korea.

Rida T. Farouki, University of California, Davis, U.S.

Spatial motion of a rigid body can be described by the trajectory curve of a reference point (e.g. the center of mass), and the moving frame along the curve. In many contexts, the adapted moving frame, whose first vector is the unit curve tangent, is desirable. The other two vectors should be the basis of the normal plane, which has a one dimensional degree of freedom. Different choices of these two normal vectors produce various adapted frames. We here address the problem of constructing rational adapted frames along a priori determined trajectory curve. If the trajectory curve is expressed as a spatial Pythagorean hodograph (PH) curve, it has a rational adapted frame known as the Euler-Rodrigues frame (ERF). By applying rationally parametrized rotation to the ERF, we can construct other rational adapted frames. When the initial and the final orientations of the frame are given, we can set up a Hermite type interpolation problem. Among the Hermite interpolants, a minimal twist frame (MTF) has the monotone twist and the least possible total twist value. The MTF with the constant angular rotation speed may be an ideal one but it is not rational in general. For the spatial quintic PH curves, the rational MTF whose angular speed function is close to a constant can be constructed by a rational quartic normal-plane rotation of the ERF. This construction method for the rational adapted frame can be applied to more general problem for the interpolation of orientation data along the PH curve. If a discrete set of orientation data is provided at nodal points on the PH curve, we express them in terms of the phase angles relative to the ERF to resolve the potential ambiguity of the data. Based on this representation, a C^1 rational spline frame can be constructed by computing the Hermite interpolants on each segment locally. For the continuity of the angular acceleration, a C^2 spline frame can be constructed by using the complex-valued cubic spline that interpolates the complex exponentials of the phase angles. An interesting case arises when the trajectory curve is a closed loop since many rigid motions in practical applications such as robotics are periodic motions that return to the same position and orientation repeatedly. Since the rotation minimizing frame starting from an initial position does not return to the same position in general, the periodic frame should have appropriate total twist. Moreover, if the closed loop is given by a PH spline curve with multiple segments, then its ERF may not have enough regularity. The normal-plane rotation for this case should be chosen carefully to achieve the regular rational adapted frame.

MS9.4 Prescribing derivative vectors of trajectories of rational motions

Wed, h. 17:35–18:00 in JRS Centre 1420

Zbynek Sir¹, Charles University in Prague.

We study the problem of constructing parametric rational motions in such a way that its trajectories match prescribed first and second derivative vectors at some points. We in particular focus on the motions associated with the polynomial and rational Pythagorean-hodograph curves. We will show that for two points this problem can be solved explicitly using polynomial PH curves of degree nine. We will study other configurations and analyze the associated quaternion equations.



Contributed and Invited Journal Presentations

CP1 Contributed session 1

Mon, h. 13:30-14:50 in JRS Centre 1420

Chair: Yongjie Jessica Zhang, Carnegie Mellon University, U.S.

CP1.1 Geometrical design of new auxetic 3D printable piezoelectric composite materials

Mon, h. 13:30-13:50 in JRS Centre 1420

Jagdish Krishnaswamy[✉], Wilfrid Laurier University, Canada.

Federico C. Buroni, University of Seville, Spain.

Felipe Garcia-Sanchez, University of Malaga, Spain.

Roderick Melnik, Wilfrid Laurier University, Canada.

Luis Rodriguez-Tembleque, University of Seville, Spain.

Andres Saez, University of Seville, Spain.

3D printing has evolved as a promising manufacturing means to fabricate multiscale and auxetic materials by allowing a fine control over the micro-structured geometrical details [1]. An optimized micro-structured geometry can offer significant control over the effective properties of the macroscale material. Auxetic materials and structures have negative Poisson's ratio [2] and can offer unforeseen implications in the areas of transduction of mechanical energy. This is because of the complex interplay between the auxetic geometry of the material and its piezoelectric response. Here, we computationally explore the role of the micro-structured geometrical design of new piezoelectric composite materials. It is expected that the combination of optimized geometrical design and 3D printing can lead to the design and fabrication of auxetic piezoelectric materials with enhanced electro-elastic properties, important for novel applications in environmentally-friendly technologies [3]. The major focus of our effort is to use computational approaches to understand the complex relations between the micro-structured geometry of piezoelectric composites and the effective electro-elastic properties of the resulting auxetic materials.

References:

- [1] F. Agnelli, A. Constantinescu, and G. Nika, "Optimal design of auxetic, additively manufactured, polymeric structures," arXiv preprint arXiv:1809.02467, 2018.
- [2] J. Dagdelen, J. Montoya, M. de Jong, and K. Persson, "Computational prediction of new auxetic materials," *Nature Communications*, vol. 8, p. 323, 2017.
- [3] A. K. Jagdish, F. C. Buroni, F. Garcia-Sanchez, R. Melnik, L. Rodriguez-Tembleque, A. Saez, "Improving the performance of lead-free piezoelectric composites by using poly-crystalline inclusions and tuning the dielectric matrix environment" (under submission).

CPI.2 Parallelized Neighbor Search with the Neighborhood Grid: A Computational Geometry Viewpoint

Mon, h. 13:50–14:10 in JRS Centre 1420

Martin Skrodzki , Freie Universität Berlin, Germany.

Ulrich Reitebuch, Freie Universität Berlin, Germany.

Konrad Polthier, Freie Universität Berlin, Germany.

The neighborhood grid data structure has been introduced as a means for fast, parallelized computation of neighborhood approximations. It aims at contexts where speed is more important than an exact neighborhood relation. Given the wide availability of graphic cards including several hundreds or thousands of shaders, the ability for parallelization makes it well suited for applications like particle rendering, biological swarm models, or human crowd motion simulations. The data structure follows a simple building procedure. It takes a point set from some finite d -dimensional space and places each of the points in a separate cell of a cubical d -dimensional grid. Within this grid, a building algorithm sorts the cells lexicographically such that along any specific dimension neighboring cells grow monotonically when considering the entry of the included point in the dimension currently considered. The grid has reached a "stable state" when the above order relation is satisfied by all neighboring cells of the grid. When introducing the data structure, the authors gave an algorithm that iterates through all dimensions and performs a single step of a bubble sort by exchanging neighboring cells that violate the above condition. However, the authors did not prove convergence of their algorithm. In the talk, we will present this proof. More generally, we prove both an upper and a lower bound of $O(n \log(n))$ for any comparison-based building algorithm acting on n points. By providing such an algorithm, we also show its time-optimality. A previously published proof of this statement depended on the assumption that for any set of n points, a unique stable state exists. We show this assumption to be correct for n up to 9. However, we also prove it wrong for any larger n . Note that the popular data structure of k - d trees also has a building time of $O(n \log(n))$. However, at this point, parallelization plays its important role. Even with massive computing power, given n processors for n points, there is no algorithm known which builds $k - d$ trees in strictly less than $O(\log(n)^2)$. But given the same resources, the neighborhood grid needs building time of only $O(\log(n))$. From a stable state of the neighborhood grid, neighborhood relations follow immediately. Given some point p in some cell in the grid, we declare all points in neighboring grid cells to be neighbors of p . This operation runs in constant time which provides an immense speed-up compared to other data structures like k - d trees, where a neighborhood look-up takes time $O(\log(n))$. However, in general, the obtained neighborhood relations will not be correct. In the talk, we provide two worst-case examples: (1) two neighboring points reside at maximum distance in the neighborhood grid, (2) no two neighboring points lie within direct neighborhood relation in the neighborhood grid. The question

remains how good the neighborhood relations of the grid are on average. To this end, we provide several experimental results.

CPI.3 Volumetric Spline Parameterization for Isogeometric Analysis

Mon, h. 14:10–14:30 in JRS Centre 1420

Falai Chen[✉], University of Science and Technology of China, China.

Maodong Pan, University of Science and Technology of China, China.

Given the spline representation of the boundary of a three dimensional domain, constructing a volumetric spline parameterization of the domain with the given boundary is a fundamental problem in isogeometric analysis. In this talk, we propose a three-stage approach for constructing volumetric parameterization satisfying the above criteria. Firstly, a harmonic map is computed between a unit cube and the computational domain. Then a bijective map modeled by a max-min optimization problem is computed in a coarse-to-fine way, and an algorithm based on divide and conquer strategy is proposed to solve the optimization problem efficiently. Finally, to ensure high quality of the parameterization, the MIPS (Most Isometric Parameterizations) method is adopted to reduce the conformal distortion of the bijective map. We provide several examples to demonstrate the feasibility of our approach and to compare our approach with some state-of-the-art methods. The results show that our algorithm produces bijective parameterization with high quality even for complex domains.

CPI.4 Introduction to unstructured splines (U-splines) for use in computer-aided geometric design software

Mon, h. 14:30–14:50 in JRS Centre 1420

Luke Engvall[✉], Coreform LLC, U.S.

Steven Schmidt, Brigham Young University, U.S.

Derek Thomas, Coreform LLC, U.S.

Michael Scott, Coreform LLC, U.S.

Unstructured splines (U-splines) are a novel spline technology that can be used in both computer-aided geometric design (CAGD) and isogeometric analysis (IGA). U-splines have the potential to overcome the limitations of existing spline descriptions (such as B-splines and T-splines) by allowing for fully local control of the element size (h-refinement), polynomial degree (p-refinement), and continuity (k-refinement). This highly local control makes U spline surfaces ideal for use in CAGD to represent shape and as a basis for isogeometric analysis.

U-splines have been designed from the ground up to serve as an analysis-suitable basis for isogeometric analysis. U-spline basis functions can be represented as a collection of Bernstein polynomials through Bezier extraction, allowing for easy integration into the isogeometric paradigm. In this talk, we will first give a basic introduction to the mathematics underlying the U-spline basis. Next, we will present an overview of the algorithm used to construct the U-spline basis functions. Finally, we will conclude by presenting some use cases of U-splines, as well as discussing some outstanding challenges.

CP2 Contributed session 2

Mon, h. 13:30-14:50 in JRS Centre 1430

Chair: Ligang Liu, University of Science and Technology of China, China.

CP2.1 Dual Quaternion Surfaces

Mon, h. 13:30–13:50 in JRS Centre 1430

Haohao Wang[✉], Southeast Missouri State University, U.S.
 Ron Goldman, Rice University, U.S.

For user interfaces, we are most impressed by extrusion techniques – translating curves and planar regions along a fixed direction into 3D to form ruled structures, or rotating curves and surfaces about a fixed axis to form surfaces and solids of revolution. These extrusion methods more closely resemble additive techniques, and with a good user interface these methods can be made simple and intuitive for design. The drawback till now is that these methods are too simple: the surfaces and solids that can be constructed by these straightforward extrusion techniques are not adequate for sophisticated design. Our approach is to generalize these extrusion methods by building extrusion operators, operators that generalize translation and rotation. The operators will be represented by curves and the operands by curves and surfaces. There will be two kinds of operators: one mimicking translation, the other rotation. But these translations and rotations will not be along a single fixed direction; rather these operations will vary along the curve representing the operator. This approach leads to a very rich collection of surfaces and solids, which will require sophisticated mathematical methods to analyze. The purpose of this paper is to build a prototype of this operator-operand user interface, and to develop the mathematics necessary to analyze these new surface and solid structures. In this presentation, we will focus on the rotation operators. We shall investigate the dual quaternion surfaces given by the parametric representation $h(s;t) = f(s)g(t)f^\dagger(s)$, where f, g are space curves represented internally by dual quaternions, f^\dagger is the dual quaternion conjugate of f , and the operation is dual quaternion multiplication. Our main goal is to utilize syzygies to study dual quaternion surfaces. We will construct three special syzygies for a dual quaternion surface from the μ -basis of one of the generating space curves. In addition, we will examine many properties of dual quaternion surfaces, and compute the implicit equation from these three special syzygies.

CP2.2 On the interference problem for ellipsoids: new closed form solutions

Mon, h. 13:50–14:10 in JRS Centre 1430

Laureano Gonzalez-Vega[✉], Universidad de Cantabria, Spain.
 Jorge Caravantes, Universidad de Alcala, Spain.

The problem of detecting when two moving ellipsoids overlap is of interest to robotics, CAD/CAM, computer animation, etc., where ellipsoids are often used for modelling (and/or enclosing) the shape of the objects under consideration. By analysing symbolically the sign of the real roots of the characteristic polynomial of the pencil defined by two ellipsoids A and B given by $X^T A X = 0$ and $X^T B X = 0$ we derive a new closed formulae characterising when A and B overlap, are separate and touch each other externally. These conditions are defined by a minimal set of polynomial inequalities depending only on the entries of A and B , need only to compute the characteristic polynomial of the pencil defined by A and B , $f(\lambda) = \det(\lambda A + B)$, and do not require the computation of the intersection points between them. Compared with the best available approach dealing with this problem the new formulae involve a smaller set of polynomials and less sign conditions. As an application this characterisation provides a new approach for exact collision detection of two moving ellipsoids since the analysis of the univariate polynomials (depending on the time) in the previously mentioned formulae provides the collision events between them.

CP2.3 Rational developable surfaces

Mon, h. 14:10–14:30 in JRS Centre 1430

Leonardo Fernández Jambrina[✉], Universidad Politécnica de Madrid, Spain.
 Alicia Cantón, Universidad Politécnica de Madrid, Spain.
 Eugenia Rosado María, Universidad Politécnica de Madrid, Spain.
 María Jesús Vázquez-Gallo, Universidad Politécnica de Madrid, Spain.

In this talk we characterise rational Bézier developable surfaces using the polar forms for their boundary curves. We generalise Aumann's algorithm for Bézier developable surfaces to the rational case and show that it allows the construction of all developable surfaces with a rational edge of regression. The conclusions can be extended to rational spline surfaces as well.

References:References

1. Aumann, G., 2003. A simple algorithm for designing developable Bézier surfaces. *Comput. Aided Geom. Des.* 20, 601?619.
2. Fernández-Jambrina, L. 2017. Bézier developable surfaces. *Comput. Aided Geom. Des.* 55, 15?28.

CP2.4 L1 splines: model, computation and recent advances

Mon, h. 14:30–14:50 in JRS Centre 1430

Ziteng Wang[✉], Northern Illinois University, U.S.

Cubic splines are widely used for data interpolation and approximation in non-parametric regression, terrain surface fitting, computer aided design, numerical control, finance, healthcare and bioinformatics. Conventional splines are constructed by minimizing the L2-norm based metrics of data fitting errors and the curvature of the spline. Such splines often show undesired oscillation and do not preserve shape, especially for irregular or multiscale data. L1 splines, by minimization of the L1-norm based metrics, have shown superior and robust shape-preserving performances and enjoyed increasing application potentials. We introduce the development of L1 splines over the past decade, present the latest research on the fast computing strategy and the quantitative measure of shape-preserving capability, and discuss future opportunities.

CP3 Contributed session 3

Mon, h. 14:50-15:50 in 2 in JRS Centre 1430

Chair: Helmut Pottmann, Technische Universität Wien, Austria.

CP3.1 Variational Convergence of Discrete Elasticae

Mon, h. 14:50–15:10 in JRS Centre 1430

Sebastian Scholtes, formerly RWTH Aachen University, Germany.
 Henrik Schumacher[✉], RWTH Aachen University, Germany.
 Max Wardetzky, University Göttingen, Germany.

We discuss a discretization by polygonal lines of the Euler-Bernoulli bending energy and of Euler elasticae under clamped boundary conditions. We show Hausdorff convergence of the set of almost minimizers of the discrete bending energy to the set of smooth Euler elasticae under mesh refinement (i) in the $W^{1,\infty}$ -topology for piecewise-linear interpolation and (ii) – utilizing a suitable smoothing operator order to create $W^{2,p}$ -curves from polygons – in the $W^{2,p}$ -topology for all $p \in [2, \infty[$.

CP3.2 Quartic Bézier curves with rational offsets Mon, h. 15:10–15:30 in JRS Centre 1430

Kai Hormann[✉], Università della Svizzera italiana, Switzerland.
Jianmin Zheng, Nanyang Technological University, Singapore.

This talk is about planar properly-parameterized, regular quartic Bézier curves that have rational offsets. Such curves are either Pythagorean hodograph (PH) or indirect Pythagorean hodograph (iPH) curves, and they include all quadratic curves, cubic PH curves, and cubic iPH curves as special cases. We give a complete analysis of these curves and derive their algebraic and geometric characterizations. The characterizations are given in terms of quantities related to the Bézier control polygon of the curves. Based on the derived characterizations, several geometric construction algorithms using quartic curves with rational offsets are presented as applications, in particular C^1 Hermite interpolation.

CP3.3 Deformation of Pythagorean hodograph curves using rectifying control polygons

Mon, h. 15:30–15:50 in JRS Centre 1430

Soo Hyun Kim[✉], Dongguk University-Seoul, South Korea.
Hwan Pyo Moon, Dongguk University-Seoul, South Korea.

Pythagorean hodograph (PH) curves are a special type of polynomial curves that their speed functions are also polynomials. Suppose we are given a PH curve and want to modify its shape for design purpose. A deformation of a polynomial curve can be easily performed by modifying its Bezier control polygon. However, the Bezier polygon is not suitable to control the shape of PH curves since any slight change of a Bezier point results in the violation of the PH condition. This is because the set of PH curves forms a low dimensional subspace in the space of polynomial curves. So we introduce the rectifying control polygon, which has (i) the end point interpolation property, (ii) the rectifying property, which means the length of the polygon is the same as the arc length of the PH curve, and (iii) the same degree of freedom as the PH curve. As a particular instance, we define the Gauss-Legendre polygon of a PH curve as the collection of edges obtained by evaluating the hodograph at the nodes of the Gauss-Legendre quadrature. Since the Gauss-Legendre quadrature evaluates the exact integral of the polynomials of fixed degree with the minimum number of nodes, the Gauss-Legendre polygon satisfies the first two properties of the rectifying polygon if it has enough number of edges compared to the degree of the PH curve. For the third property about the degree of freedom, the situation depends on the dimension of the space. For any planar PH curve of degree $2n + 1$, the Gauss-Legendre polygon of $n + 1$ edges becomes the rectifying control polygon. Conversely, for a given Gauss-Legendre polygon of $n + 1$ edges, we can compute multiple instances of PH curves of degree $2n + 1$. For the spatial PH curves, if the degree is $6m + 1$, then the Gauss-Legendre polygon with $4m + 1$ edges becomes the rectifying control polygon. The first nontrivial case arises the Gauss-Legendre polygon with 5 edges for the spatial septic PH curves. We present the method to compute the spatial septic PH curves from the given Gauss-Legendre polygon. A drawback of the Gauss-Legendre polygon is that it does not determine the end tangents of the PH curve. As an alternative to the Gauss-Legendre polygon, we introduce the Gauss-Lobatto polygon. The Gauss-Lobatto polygon naturally determines the end tangent vectors since the Gauss-Lobatto quadrature has the end points of the parameter domain as the predetermined nodes. The Gauss-Lobatto polygon with enough number of edges become the rectifying polygon

as well. However, the property on the degree of freedom is quite subtle. Unfortunately, planar PH curves do not allow the Gauss-Lobatto rectifying control polygon. For the spatial case, PH curves of degree $6m + 1$ have the Gauss-Lobatto rectifying control polygon with $4m + 1$ edges. We can also compute the spatial septic PH curves from the given Gauss-Lobatto polygon, and develop the algorithm to deform the PH curve using its Gauss-Lobatto polygon. This algorithm is very useful especially for the deformation of PH spline curves.

CP4 Contributed session 4

Tue, h. 9:00-10:00 in in JRS Centre 1420

Chair: Michael Scott, Brigham Young University, U.S.

CP4.1 Rational Offsets of Regular Quadrics Revisited *(Invited CAGD 2018 Paper)*

Tue, h. 9:00-9:20 in in JRS Centre 1420

Severinas Zube[✉], Vilnius University, Lithuania
Rimvydas Krasauskas, Vilnius University, Lithuania

New lower degree rational parametrizations of regular quadric offsets in the 3-dimensional Euclidean space are explicitly derived. Offsets of ellipsoids, elliptic paraboloids, and doubly ruled quadrics are parametrized with bidegrees (4, 8), (4, 6), and (4, 4), respectively. These degree bounds are expected to be minimal. For the parametrization we use the dual surface: oriented tangent planes of a given quadric in \mathbb{R}^3 define a surface in the Blaschke cylinder, which is actually an isotropic cyclide. The dual parametrization of the isotropic cyclide gives the parametrization of the quadric offset. The most complicated cases of such cyclides have two real components. Therefore, R-birational parametrization is not possible, and one can hope to parametrize both components separately. The required parametrizations were described in Quaternionic-Bézier formulas, where control points and weights are elements in Quaternion algebra. There is a usual way for the converting these parametrization formulas in Bernstein polynomial basis.

CP4.2 Certified space curve fitting and trajectory planning for CNC machining with cubic B-splines *(Invited CAD 2019 Paper)*

Tue, h. 9:20-9:40 in JRS Centre 1420

Fengming Lin, School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing, China.

Li-Yong Shen[✉], School of Mathematical Sciences, University of CAS, Beijing, China.

Chun-Ming Yuan, KLMM, Academy of Mathematics and Systems Sciences, CAS, Beijing, China.

Zhenpeng Mi, KLMM, Academy of Mathematics and Systems Sciences, CAS, Beijing, China.

In CNC machining, the tool path following G01 codes generally introduces large computations and inherent discontinuities. A common way to avoid these shortcomings is fitting the G01 polyline with a parametric curve and then scheduling the feedrate along the fitted curve. However, curve fitting with confined error in three dimensional space is nontrivial since the Hausdorff distance between a space G01 segment and a rational parametric curve segment is difficult to formulate. In this paper, we derive the explicit expression for the Hausdorff distance between a line segment and a curve segment, and then propose a curve fitting algorithm for G01 polylines. Instead of the traditional two-stage model, we present a combined trajectory planning model

with error constraints as well as dynamic constraints. Moreover, an effective and efficient algorithm is designed to solve this model. Experimental results are provided to illustrate and clarify our methods.

CP4.3 Optimizing B-spline surfaces for developability and paneling architectural freeform surfaces *(Invited CAD 2019 Paper)* Tue, h. 9:40–10:00 in JRS Centre 1420

Konstantinos Gavriil, Evolute GmbH, Austria.
Alexander Schiffner, Evolute GmbH, Austria.
Helmut Pottmann[✉], KAUST, Saudi Arabia.

Motivated by applications in architecture and design, we present a novel method for increasing the developability of a B-spline surface. We use the property that the Gauss image of a developable surface is 1-dimensional and can be locally well approximated by circles. This is cast into an algorithm for thinning the Gauss image by increasing the planarity of the Gauss images of appropriate neighborhoods. A variation of the main method allows us to tackle the problem of paneling a freeform architectural surface with developable panels, in particular enforcing rotational cylindrical, rotational conical and planar panels, which are the main preferred types of developable panels in architecture due to the reduced cost of manufacturing.

CP5 Contributed session 5 Tue, h. 9:00–10:00 in JRS Centre 1430

Chair: Dan Gonsor, The Boeing Company, U.S.

CP5.1 Customization and Topology Optimization of Compression Casts/Braces on Two-Manifold Surfaces *(Invited CAD 2019 Paper)* Tue, h. 9:00–9:20 in JRS Centre 1430

Yunbo Zhang[✉], Rochester Institute of Technology, U.S.
Tsz-Ho Kwok, Concordia University, Canada.

This paper applies the topology optimization (TO) technique to the design of custom compression casts/braces on two-manifold mesh surfaces. Conventional braces or casts, usually made of plaster or fiberglass, have the drawbacks of being heavy and unventilated to wear. To reduce the weight and improve the performance of a custom brace, TO methods are adopted to optimize the geometry of the brace in the three-dimensional (3D) space, but they are computationally expensive. Based on our observation that the brace has a much smaller thickness compared to other dimensions and the applied loads are normal forces, this paper presents a novel TO method based on thin plate elements on the two-dimensional manifold (2-manifold) surfaces instead of 3D solid elements. Our working pipeline starts from a 3D scan of a human body represented by a 2-manifold mesh surface, which is the base design domain for the custom brace. Similar to the concept of isoparametric representation, the 3D design domain is mapped onto a two-dimensional (2D) parametric domain. A Finite Element Analysis (FEA) with bending moments is performed on the parameterized 2D design domain, and the Solid Isotropic Material with Penalization (SIMP) method is applied to optimize the pattern in the parametric domain. After the optimized cast/brace is obtained on the 2-manifold mesh surface, a solid model is generated by our design interface and then sent to a 3D printer for fabrication. Compared with the optimization method with solid elements, our method is more efficient and controllable due to the high efficiency of solving FEA in the 2D domain.

CP5.2 Automatic and high-quality surface mesh generation for CAD models (*Invited CAD 2019 Paper*) Tue, h. 9:20–9:40 in JRS Centre 1430

Jianwei Guo[✉], NLPR, Institute of Automation, Chinese Academy of Sciences, China.

Xiaohong Jia, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China.

Dong-Ming Yan, NLPR, Institute of Automation, Chinese Academy of Sciences, China.

In this paper, we present a fully automatic framework that tessellates industrial computer-aided design (CAD) models into high-quality triangular meshes. In contrast to previous approaches that are purely parametric or performed directly in 3D space, our method is based on a remeshing algorithm that can achieve accuracy and high-quality simultaneously. Given an input standard CAD model, which is represented by B-rep format, we first rebuild the parametric domain for each surface patch according to an initial triangulation, and then the boundaries of the parametric domain are retriangulated by exploiting the Constrained Delaunay Triangulation (CDT). In the second stage, the 2D triangulation is projected back to the 3D space, and a modified global isotropic remeshing process is applied, which further improves the regularity and angle quality of the tessellated meshes. Experiments demonstrate the validity of the proposed approach and its ability to generate high-quality meshes. Moreover, we evaluate our technique and compare it with state-of-the-art CAD model tessellation approaches.

CP5.3 Adaptive Slicing Based on Efficient Profile Analysis (*Invited CAD 2019 Paper*) Tue, h. 9:40–10:00 in JRS Centre 1430

Huachao Mao, University of Southern California, U.S.

Tsz Ho Kwok[✉], Concordia University, Canada.

Yong Chen, University of Southern California, U.S.

Charlie C. L. Wang, The Chinese University of Hong Kong, Hong Kong.

Adaptive slicing is an important computational task required in the layer-based manufacturing process. Its purpose is to find an optimal trade-off between the fabrication time (number of layers) and the surface quality (geometric deviation error). Most of the traditional adaptive slicing algorithms are computationally expensive or only based on local evaluation of errors. To tackle these problems, we introduce a method to efficiently generate the slicing plans by a new metric profile that can characterize the distribution of deviation errors along the building direction. By generalizing the conventional error metrics, the proposed metric profile is a density function of deviation errors, which measures the global deviation errors rather than the in-plane local geometry errors used in most prior methods. Slicing can be efficiently evaluated based on metric profiles in contrast to the expensive computation on models in boundary-representation. An efficient algorithm based on dynamic programming is proposed to find the best slicing plan. Our adaptive slicing method can also be applied to models with weighted features and can serve as the inner loop to search the best building direction. The performance of our approach is demonstrated by experimental tests on different examples.

CP6 Contributed session 6 Tue, h. 15:50–16:30 in JRS Centre 1430

Chair: Stefanie Hahmann, University Grenoble, France.

CP6.1 Support-free Elliptic Hollowing for 3D Printing via the Voronoi Diagram of Ellipses*(Invited CAD 2018 Paper)*

Tue, h. 15:50–16:10 in JRS Centre 1430

Mokwon Lee , Voronoi Diagram Research Center, Hanyang University, South Korea.

Qing Fang, School of Mathematical Science, University of Science and Technology of China,

Youngsong Cho, Voronoi Diagram Research Center, Hanyang University, South Korea.

Joonghyun Ryu, Voronoi Diagram Research Center, Hanyang University, South Korea.

Ligang Liu, School of Mathematical Science, University of Science and Technology of China, China.

Deok-Soo Kim, School of Mechanical Engineering/Voronoi Diagram Research Center, Hanyang University, South Korea.

Abstract: 3D printing has become very popular. As it gets more popular, printing efficiency would be more critical. To print a product faster, cheaper, and lighter, the reduction of the volume to print is one of the key factors and thus many studies for hollowing 3D objects have been conducted. In this talk, we will present an algorithm (together with its implementation and demonstration) to generate elliptic hollows inside of 3D shape model so that it is free from any additional supporting structure. To generate elliptic hollows, we propose two approaches. In the first approach, we generate and pack elliptic voids within a 2D polygon defined by the section plane cutting through the object. Then we project the planar packing to the space in the proximity using nearby parallel cross-sections. We use the Voronoi diagram of ellipses in the polygon for efficiently packing ellipses [See Lee et al. Support-free hollowing for 3D Print via Voronoi Diagram of Ellipses, Computer-Aided Design, 2018]. In the second approach, we directly use the Voronoi diagram of ellipsoids in the 3D polyherdral object to pack ellipsoidal hollows in the object. We demonstrate the proposed idea using the approximated Voronoi diagram computed using the Voronoi diagram of 3D spheres.

CP6.2 Curvature adaptive surface remeshing by sampling normal cycle *(Invited CAD 2019**Paper)*

Tue, h. 16:10–16:30 in JRS Centre 1430

Kehua Su, Wuhan University, China.

Na Lei, Dalian University of Technology, China.

Wei Chen, Dalian University of Technology, China.

Li Cui, Beijing Normal University, China.

Hang Si, WIAS, Germany.

Shikui Chen, Stony Brook University, U.S.

Xianfeng Gu , Stony Brook University, U.S.

Surface meshing plays a fundamental important role in Visualization and Computer Graphics, which produces discrete meshes to approximate a smooth surface. Many geometric processing tasks heavily depend on the qualities of the meshes, especially the convergence in terms of topology, position, Riemannian metric, differential operators and curvature measures. Normal cycle theory points out that in order to guarantee the convergence of curvature measures, the discrete meshes are required to approximate not only the smooth surface itself, but also the normal cycle of the surface. This theory inspires the development of the remeshing method based on conformal parameterization and planar Delaunay refinement, which uniformly samples the smooth surface, and produces Delaunay triangulations with bounded minimal corner angles. This method ensures the Hausdorff distances between the normal cycles of the resulting meshes and the smooth normal cycle converges to 0, the discrete Gaussian curvature

and mean curvature measures of the resulting meshes converge to their counter parts on the smooth surface. In the current work, the conformal parameterization based remeshing algorithm is further improved to speed up the curvature convergence. Instead of uniformly sampling the surface itself, the novel algorithm samples the normal cycle of the surface. The algorithm pipeline is as follows: first, two parameterizations are constructed, one is the surface conformal parameterization based on dynamic Ricci flow, the other is the normal cycle area-preserving parameterization based on optimal mass transportation; second, the normal cycle parameterization is uniformly sampled; third, the Delaunay refinement mesh generation is carried out on the surface conformal parameterization. The produced meshes can be proven to converge to the smooth surface in terms of curvature measures. Experimental results demonstrate the efficiency and efficacy of proposed algorithm, the convergence speeds of the curvatures are prominently faster than those of conventional methods.

CP7 Contributed session 7

Wed, h. 16:20–17:40 in JRS Centre 1430

Chair: Bert Jüttler, Johannes Kepler Universität Linz, Austria

CP7.1 Discrete surfaces in sphere geometries

Wed, h. 16:20–16:40 in JRS Centre 1430

Helmut Pottmann[✉], Technische Universität Wien, Austria.

Discrete surfaces play a central role in numerous applications and constitute an active area of mathematical research. Some of these discrete surface representations are essentially objects of sphere geometries. Examples include the most prominent discrete versions of principal surface parameterizations, namely circular and conical meshes, which belong to Möbius and Laguerre sphere geometry, respectively. In this talk, we briefly present results of ongoing research on various types of discrete surfaces within sphere geometries. These include new discretizations of surfaces which exhibit a linear relation between their principal curvatures and which possess potential applications in architectural geometry. This is joint work with: M. Kilian, C. Müller, D. Pellis, J. Wallner and H. Wang.

CP7.2 Bernstein-Bézier techniques for piecewise harmonic polynomials.

Wed, h. 16:40–17:00 in JRS Centre 1430

Tatyana Sorokina[✉], Towson University, U.S.

Snangyou Zhang, University of Delaware, U.S.

Bernstein-Bézier techniques for analyzing continuous piecewise harmonic polynomials in n variables are developed. Dimension formulae and minimal determining sets for several important spaces are obtained using the new techniques. Based on these results, two new conforming quadratic harmonic polynomial finite elements are designed on special triangular grids. The optimal order of convergence is proved for both finite elements, and confirmed by numerical computations. In addition, numerical comparisons with the standard conforming finite elements are presented.

CP7.3 Shape preserving properties of weighted f-transformed systems

Wed, h. 17:00–17:20 in JRS Centre 1430

Esmeralda Mainar, University of Zaragoza, Spain.

Juan Manuel Peña, University of Zaragoza, Spain.

Beatriz Rubio[✉], University of Zaragoza, Spain.

Given a system of functions, we introduce the concept of weighted f -transformed system, which will include a very large class of useful representations in computer-aided design. A weighted f -transformed system inherits, from the initial system, shape preserving properties of the corresponding representations. Computational methods for the accurate resolution of algebraic problems with collocation matrices of weighted f -transformed systems will be presented.

CP7.4 Convergence rates of iterative rational Hermite interpolants

Wed, h. 17:20–17:40 in JRS Centre 1430

Emiliano Cirillo , Università della Svizzera italiana, Switzerland.

Kai Hormann, Università della Svizzera italiana, Switzerland.

Jean Sidon, Israel.

Cirillo and Hormann introduce an iterative approach to the Hermite interpolation problem, which, starting from the Lagrange interpolant, successively adds m corrections terms to interpolate the data up to the m -th derivative. The method is general enough to be applied to any interpolant in linear form with a sufficiently continuous set of basis functions, but Cirillo and Hormann focus their attention on Floater–Hormann interpolants, a family of barycentric rational interpolants that are based on a particular blend of local polynomial interpolants of degree d . They show that the resulting iterative rational Hermite interpolants converge at the rate of $O(h^{(m+1)(d+1)})$ as the mesh size h converges to zero for $m = 1, 2$, and their numerical results suggest that the same rate holds for $m > 2$. In this talk we show that the same convergence rate holds for any $m \geq 1$.



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